LINEAR PROGRAMMING

Solve the following inequalities;

1. $3x - 5 > -2$

2. $\frac{3}{2}x - \frac{2}{3}(1-2x) < 5$

NB: When solving inequalities we need to take note of the following;

(i) DON'T replace the inequality sign with an equal sign.

(ii) When dividing by a negative, the inequality sign MUST change.

Finding the least and greatest integral values in a given inequality.

An integral value is a whole number and the required values can be found by representing our solution on a number line.

Eg. Find the greatest integral value for which

$2x - 7 < x - 2$

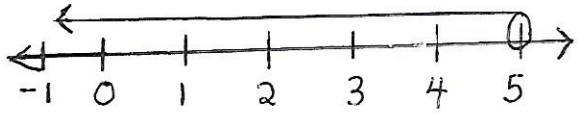
Solution:

First solve the inequality as shown below.

$2x - 7 < x - 2$

$x < 5$

then we use a number line to show our solution



Since x can't be equal to 5, the greatest integral value is 4

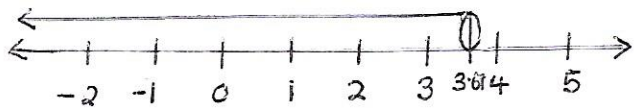
Eg 2. Find the greatest integral value of x if

$5 - 2x > x - 6$

$-2x - x > -6 - 5$

$-3x > -11$

$x < \frac{11}{3}$ or $x < 3 \cdot 67$



$\therefore x = 3$.

Eg 3. Find the integral values that satisfy

$7 \geq 4 - 3x > -3$,

first split the inequalities as shown below

$7 \geq 4 - 3x$ and $4 - 3x > -3$

then solve the inequalities

$7 \geq 4 - 3x$ and $4 - 3x > -3$

$3x \geq -3$ $\quad -3x > -7$

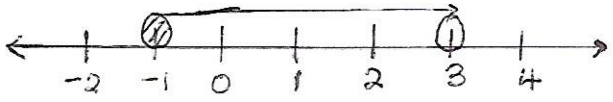
$x \geq -1$ $\quad x < \frac{7}{3}$

or $x < 2 \cdot 33$

Then combine the inequalities as shown below

$-1 \leq x < 2 \cdot 33$

Using a number line



$\therefore$ the integral values are $\{-1, 0, 1, 2\}$.

Exercise

1. Write down the least integral value of x which satisfies the following;

a) $9 - 2x < 4$

b) $2x - 3 \geq 5$

c) $4 - 3x \leq 8$

2. Find the integral values that satisfy

$7 < 3x + 1 < 19$

Forming inequalities

The following words are equivalent to the inequality signs given below.

At least — $\geq$

More than — $>$

Not more than — $\leq$

At most — $\leq$

Does not exceed — $\leq$

Write down the inequalities representing the following statements;

a) A man wants to buy a car but he must have at least 20 million shillings.

Let the amount of money be x

then $x \geq 20,000,000$

b) The total cost of x books at shs. 500 each and y pencils at shs. 300 each is not more than shs. 20000.

If 1 book costs shs. 500
x books cost shs. $500x$

1 pencil costs shs. 300
y Pencil cost shs. $300y$

The total cost is $500x + 300y$

$\therefore 500x + 300y \leq 20000$

then reduce the inequality

$$\frac{500}{100}x + \frac{300}{100}y \leq \frac{20000}{100}$$

$$= 5x + 3y \leq 200$$

c) A student bought p books at shs. 750 each and t pens at shs. 500 each. He spent more than shs. 9000

$$\frac{750}{250}p + \frac{500}{250}t > \frac{9000}{250}$$

$$= 3p + 2t > 36$$

Example 1

Mrs. Mukasa is going to bake chocolate cakes and yellow cakes for Sale. She wants to bake at least 2 chocolate cakes. She also wants to bake more yellow cakes than chocolate cakes. Due to limited time and facilities she cannot bake more than 10 cakes.

The chocolate cakes are to be sold at shs. 1500 and the yellow cakes are to be sold at shs. 1000. To make a profit, more than shs. 8000 must be realised from the sales.

(i) How many cakes of each type should Mrs. Mukasa bake in order to make maximum profit?

(ii) What is the minimum number of cakes she can bake and still make a profit?

Solution

Let the number of chocolate cakes be x and the number of yellow cakes be y.

Identify the inequalities from the question above.

(i) the number of chocolate cakes

$x \geq 2$

(ii) More ~~chocolate~~ yellow cakes than chocolate cakes

$y > x$

(iii) The total number of cakes she can bake

$x + y \leq 10$

(iv) To make a profit

$$\frac{1500x}{500} + \frac{1000y}{500} > \frac{8000}{500}$$

$$3x + 2y > 16$$

The inequalities from the question are

(i) $x \geq 2$

(ii) $y > x$

(iii) $x + y \leq 10$

(iv) $3x + 2y > 16$

Next represent your inequalities on a graph and then shade the unwanted regions to show the feasible region.

The lines to be plotted are

$x = 2$

$y = x$

$x + y = 10$

$3x + 2y = 16$

BUT $y = x$ and $3x + 2y = 16$ are dotted lines because there was no equal sign below the inequality sign.

For $x + y = 10$

x	0	10
y	10	0

use (0,10) and (10,0)

for $3x + 2y = 16$

x	0	5·3
y	8	0

use (0,8) and (5·3,0)

i) To find out the maximum profit she will get we identify the integral values in the feasible region which are any values near the top or at the border of the feasible region ie (2,8), (3,7), (4,6) and find out how much will be got.

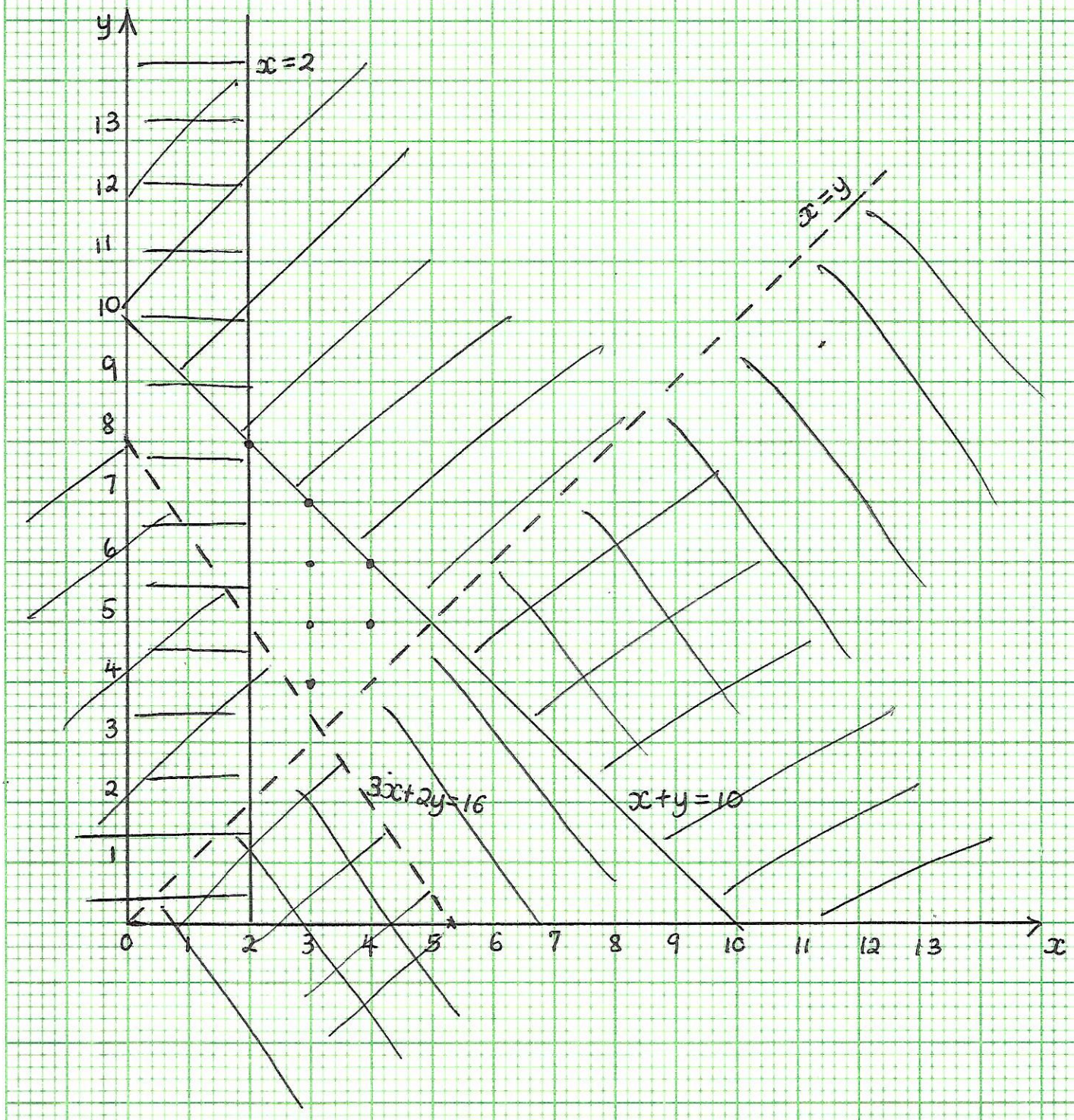
(2,8) = 2x1500 + 8x1000 = 11,000

(3,7) = 3x1500 + 7x1000 = 11,500

(4,6) = 4x1500 + 6x1000 = 12,000

∴ Mrs Mukasa should bake 4 chocolate cakes and 6 yellow cakes to make maximum profit.

Example 1



(ii) What is the minimum number of cakes she can bake and still make a profit?

Look for the integral values in the lower part of your feasible region ie (3,4) and (3,5)

$(3,4) = 3\times1500 + 4\times1000 = 8500$

$(3,5) = 3\times1500 + 5\times1000 = 9500$

Since to make a profit, the sales must be more than shs. 8000, the minimum number of cakes she will bake are 3 chocolate cakes and 4 yellow cakes.

Example 2

A school constructed an office block which required 34 tonnes of sand. The school hired a lorry and tipper truck with capacities of 7 tonnes and 5 tonnes respectively to transport the sand. The cost per trip either by lorry or tipper truck was shs. 30,000. The money available for transportation was shs. 180,000. The trips made by the lorry did not exceed those made by the tipper truck.

a) If x and y represent the number of trips made by the lorry and the tipper truck respectively;

(i) Write down five inequalities to represent the given information.

Inequality for the tonnage

$7x + 5y \geq 34$

This is because the tonnes of sand needed can't be less than 34

Inequality for cost

$$\frac{30{,}000x}{30000} + \frac{30000y}{30000} \leq \frac{180000}{30000}$$

$$x + y \leq 6$$

Inequality for no. of trips

$$x \leq y$$

The other two inequalities will be

$$x \geq 0$$

$$y \geq 0$$

Showing the possibility that the lorry and the tipper truck were hired to transport the sand.

∴ The inequalities are

$$7x + 5y \geq 34$$

$$x + y \leq 6$$

$$x \leq y$$

$$x \geq 0$$

$$y \geq 0$$

(ii) Plot these inequalities to represent the given information.

The lines to plot are

$7x + 5y = 34$

x	0	4.9
y	6.8	0

$x + y = 6$

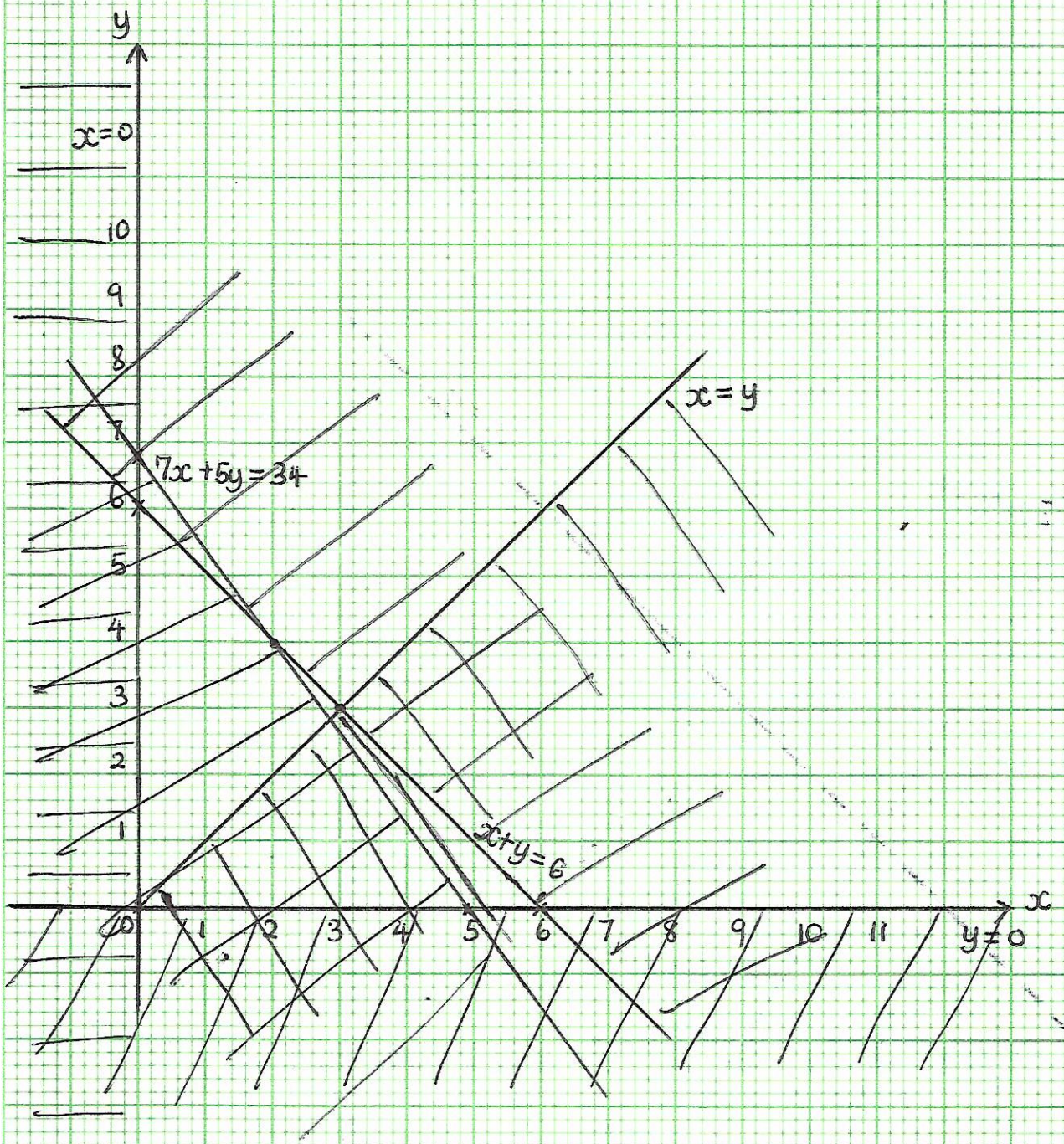
x	0	6
y	6	0

$x = y$

$x = 0$

$y = 0$

Example 2.



b(i) From your graph in a(ii) above list all the possible number of trips that each vehicle can make so as to maximise total tonnage of sand transported.

(2, 4)

(3, 3)

The lorry can make 2 trips and the tipper truck 4 trips or the lorry can make 3 trips and the tipper truck 3 trips.

(ii) Find the number of trips by each vehicle that made the greatest total tonnage.

$(2,4) = 2\times7 + 4\times5$

$= 14 + 20$

$= 34$

$(3,3) = 3\times7 + 3\times5$

$= 21 + 15$

$= 36$

The greatest total tonnage was made by 3 trips of the lorry and 3 trips of the tipper truck.

Example 3

A school hired a bus and a minibus to transport students to a study tour. Each trip by the bus cost shs. 40,000 and that of the minibus cost shs. 25000. The bus has a capacity of 42 students and the minibus 14 students. All the 126 students contributed a total of shs. 200,000 and had to go for the tour. The minibus had to make more trips than the bus. If x and y represent the number of trips made by the bus and the minibus respectively;

a) Write down 5 inequalities representing the above

Inequality for the cost

$$\frac{40000x}{5000} + \frac{25000y}{5000} \leq \frac{200,000}{5000}$$

$= 8x + 5y \leq 40$

Inequality for the no. of students

$$\frac{42x}{14} + \frac{14y}{14} \geq \frac{126}{14}$$

NB We use $\geq$ because all the students had to go

$= 3x + y \geq 9$

Inequality for the no. of trip's

$x < y$ or $y > x$

The other 2 inequalities are

$x \geq 0$

$y \geq 0$ Showing that the bus and minibus were hired to transport the students.

∴ The inequalities are

$8x + 5y \leq 40$

$3x + y \geq 9$

$x < y$

$x \geq 0$

$y \geq 0$

b(i) Plot the inequalities on the same axes.

The lines to be plotted are

$8x + 5y = 40$

$3x + y = 9$

$x = y$

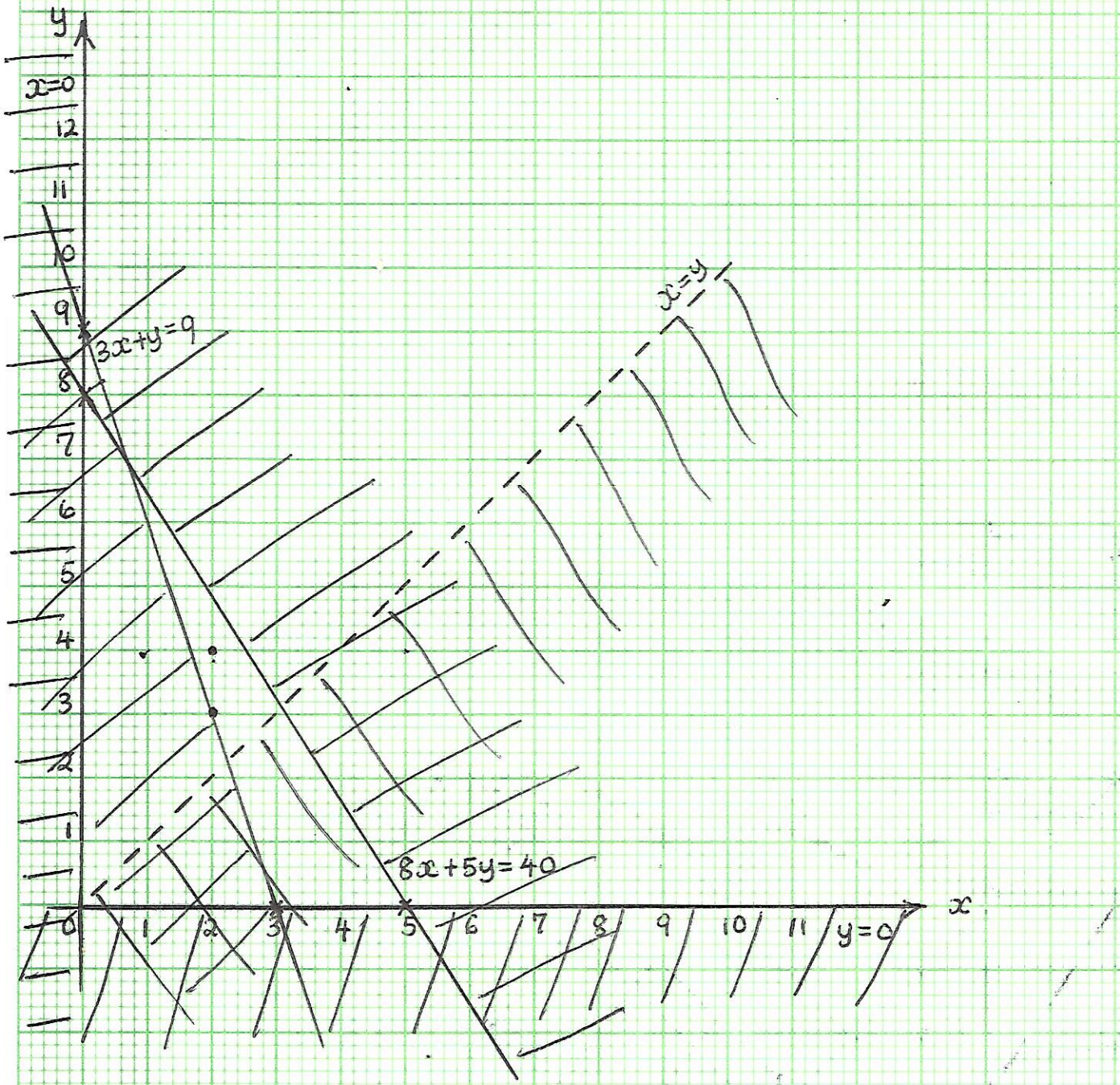
for $8x + 5y = 40$

x	0	5
y	8	0

for $3x + y = 9$

x	0	3
y	9	0

Example 3



(ii) By Shading the unwanted region, Show the region satisfying all the inequalities

c) Use the graph to find the number of trips each vehicle should make so as to spend the least amou of money.

From the feasible region our integral values are (2,3) and (2,4)

$\therefore (2,3) = 2 \times 40{,}000 + 3 \times 25000$

$= 80000 + 75000$

$= \text{Shs. } 155{,}000$

$(2,4) = 2 \times 40{,}000 + 4 \times 25000$

$= 80{,}000 + 100000$

$= \text{Shs. } 180{,}000$

$\therefore$ The bus should make 2 trips and the minibus 3 trips so as to spend the least amount of money

~~E/xe~~

Exercise

1. A bicycle factory assembles 2 types of bicycles, Roadmaster and Hero on different assembly lines. An assembly line for Roadmaster occupies an area of $60m^2$ and that of Hero occupies an area of $30m^2$ of floor space. The floor space available for all the assembly lines is $420m^2$. The assembly line for Roadmaster needs 10 men to operate it and that of Hero needs 16 men to operate it. The assembly lines need a maximum of 120 men to operate them.

a) If x and y represent the number of assembly lines for Roadmaster and Hero respectively;

(i) Form 4 inequalities to represent the given information.

(ii) Draw graphs on the same axes to represent the inequalities in (i) above and shade the unwanted regions.

b) The assembly lines for Roadmaster produce 30 bicycles per day and that of Hero produces 20 bicycles per day. Find the

(i) Number of assembly lines for each type of bicycle that should be operated so as to produce the highest total number of bicycles per day.

(ii) highest total number of bicycles that can be produced per day.

2. The manager of a cinema hall wishes to divide the seats available into 2 classes executive and ordinary. There are not more than 120 seats available. There must be at least twice as many ordinary seats as there are executive seats. Executive seats are priced at shs. 15000 each and ordinary seats are priced at shs. 10000 each. At least shs. 1,000,000 should be collected at each show to meet the expenses.

a) Taking x as the number of executive seats and y as the number of ordinary seats, write down 5 inequalities

b) Represent the inequalities on a graph

c) From your graph, find the number of seats of each kind which must be sold to give the maximum profit

3. A company wishes to transport at least 480 mattresse from its store to one of its sales point. It has 2 type of trucks, A and B. Truck A can carry 40 mattresses at a cost of shs. 30000 per trip. Truck B can carry 60 mattresses at a cost of shs. 45000 per trip. There is shs. 600,000 available for transport. The number of trips made by A should not exceed 12. Those made by B should not exceed twice the number of trips made by A.

a) If x and y are the trips made by A and B respectively, write down 4 inequalities satisfying the given condition

b) (i) On the same axes, draw the graphs of the inequalities and shade the unwanted regions.

(ii) Find all the possible number of trips made by each truck so that the transport cost is minimised.

4. A farmer plans to plant an 18 hectare field with carrots and potatoes. The farmer's estimates for the project are shown in the table below.

	CARROTS	POTATOES
Planting and harvesting costs per hectare	shs. 95,000	shs. 60,000
Number of working hours per hectare	12 days	4 days
Expected profit per hectare	shs. 228,000	shs. 157,000

The farmer has only Shs. 1,140,000 to invest in the project. The total number of working days is 120.

a) By letting x represent the number of hectares to be planted with carrots and y the number of hectares to be planted with potatoes, write down the inequalities for

(i) Cost of the project

(ii) Working days

(iii) Number of hectares used in the project

(iv) The possibility that the field will at least be used in planting either carrots or potatoes.

b) Write down the expression for the profit P, in terms of x and y.

c(i) On the same axes plot graphs of the inequalities in a) above and shade the unwanted regions.

(ii) Use your graphs to determine how the farmer should use the field to maximise profit. Hence find the farmer's maximum profit.

5. At a graduation party, the guests are to be served with beer and Soda. At least twice as many crates of beer as crates of soda are needed. A crate of beer contains 25 bottles and a crate of Soda contains 24 bottles. More than 200 bottles of beer and Soda are needed. A maximum of Shs. 500,000 may be spent on beer and Soda. Assume a crate of beer costs Shs. 40,000 and that of Soda costs shs. 15000.

a(i) Form inequalities to represent the above statements.

(ii) Represent the above inequalities on the same axes.

(iii) By shading the unwanted region, represent the region satisfying the inequalities above.

b) From your graph, find the number of crates of beer and Soda that should be bought if the cost is to be as low as possible.
Find the amount that was paid for these crates of beer and Soda.

6. A trader has Shs. 250,000. He buys boxes of books at Shs. 25000 each and boxes of candles at shs. 10000 each. The money spent on books is at least shs. 50000 more than that spent on candles. He buys at least 5 boxes of books and at least 7 boxes of candles.

a) Write down four inequalities to represent this information.

b)(i) On the same axes, plot the graphs of the inequalities and shade the unwanted regions

(ii) List all the possible numbers of boxes of books and candles he can buy.

C) Find the number of boxes of books and candles that the trader should buy so as to spend all the money.